TM no. M-43-76-3

TECHNICAL MEMORANDUM

CIVIL ENGINEERING LABORATORY · CEL · PORT HUENEME, CALIF.

title: Elementary Equations of Motion for Free Ascending Cylindrical Pontoons

author: K. W. Tate

date: April 1976

sponsor: Naval Sea Systems Command

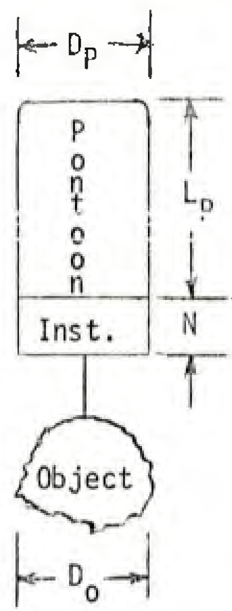
program nos: 43-023

CIVIL ENGINEERING LABORATORY
NAVAL CONSTRUCTION BATTALION CENTER
Port Hueneme, California 93043

ELEMENTARY EQUATIONS OF MOTION FOR FREE ASCENDING CYLINDRICAL PONTOONS

INTRODUCTION

The following paragraphs present several elementary equations to describe the motion of a free ascending recovery system and attached object. The recovery system consists of a buoyancy pontoon and an instrument compartment. The buoyancy pontoon is a pressure balanced vertical cylinder with a negligible mass. The instrument compartment is located immediately below the pontoon and contains the necessary instrumentation, batteries, and gas generator equipment to operate the recovery system. The instrument compartment also contains sufficient buoyancy foam to make the entire recovery system neutrally buoyant. The recovery object geometry is unspecified but assumed to be hydrodynamically stable with a maximum diameter of D_o, and cross-sectional area of A_o.



The water is a uniform density medium, initially motionless. The buoyancy gas vented from the pontoon is exhausted in such a manner as to not influence the motion of either the pontoon/object or the surrounding water column. The drag force exerted on the pontoon and object is viscous pressure drag; skin friction drag is negligible. Finally, the pontoon and object are close-coupled and behave as a single body whose characteristic dimension is the maximum projected cross-sectional area normal to the direction of motion.

The use of this simple flow model permits the interaction of geometry variables and pontoon motion to be studied without the influence of nonuniformities which may exist in the flow field such as ocean currents, or density changes. This flow model will be used to predict the ascent velocity and change in ascent velocity for free ascending pontoons and to predict porpoising of pontoons at the surface. Dimensionless parameters will be used wherever practical to present generalized results.

ASCENT VELOCITY

Based on the assumptions listed above, the ascent velocity of the pontoon/object is

$$\dot{X} = \left(\frac{2\ F_B}{\rho_W C_D A_D}\right)^{1/2} \tag{1}$$

where $\dot{X}$ = ascent velocity

F_B = net system positive buoyancy

A_D = maximum projected cross-sectional area

C_D = empirical drag coefficient

ρ_W = seawater density

Recall that the weight of seawater displaced by the pontoon is

$$F_P = \rho_W A_P L_P \tag{2}$$

where F_P = weight of seawater replaced by pontoon

A_P = pontoon cross-sectional area

L_P = pontoon length

Substituting for ρ_W in equation (1) we obtain

$$\dot{X} = \left(\frac{2}{C_D}\ \frac{F_B}{F_P}\ \frac{A_P}{A_D}\ L_P\right)^{1/2} \tag{3}$$

Introducing the nondimensional terms

$$\alpha = A_P/A_D \tag{4}$$

and

$$\beta = F_B/F_P \tag{5}$$

α is the ratio of the pontoon cross-sectional area to the maximum projected cross-sectional area of the pontoon/object. β is the ratio of the system positive buoyancy to the displacement of the pontoon. Note that the range of both these parameters is zero to unity.

$$0 < \alpha, \beta \leq 1 \tag{6}$$

A value of unity for α means the pontoon cross-sectional area is greater than or equal to the object cross-sectional area. A value of α less than unity means that the object area is greater than the pontoon area. A value of unity for β means the pontoon is ascending without an object. For typical recovery operations, the value of β will be 0.1 or less.

Substituting equations (4) and (5) into (3) we obtain

$$\dot{X} = \left(\frac{2}{C_D} \beta \alpha L_p\right)^{1/2} \tag{7}$$

The drag coefficient of bluff bodies is rather insensitive to geometry, and most shapes have a drag coefficient of approximately unity. Thus, we may consider equation (7) to contain two independent variables, β and L_p, and one dependent variable, α. Consider first the case where only very dense objects are being recovered, i.e., $\alpha = 1$. Figure 1 shows the variation of ascent velocity with pontoon length and buoyancy ratio. For a constant buoyancy ratio the ascent velocity increases with increasing pontoon length. Alternately, to maintain a constant ascent velocity, the buoyancy ratio must be decreased as the pontoon length is increased. In a model study, or where a family of pontoons are to be developed concurrently, it should be noted that if a constant ascent velocity is to be used, the buoyancy ratios of the larger pontoons must be reduced. Similarly, if a constant buoyancy ratio is desired, the larger pontoons will ascend faster than the smaller ones.

In typical "large" recovery operations, the α term is expected to be less than unity. Sunken ships or missiles have large voids or flooded compartments which result in the area ratio being quite small. In this case, the ascent velocity will be lower than shown in Figure 1 for a specified buoyancy ratio. The result will be that for bulky objects, the buoyancy ratio must be increased to maintain a specified ascent rate and this extra buoyancy will be dissipated as drag on the object.

For recovery operations from moderate (5000 ft) depths, the pontoon positive buoyancy and, hence, pontoon ascent velocity are fairly constant. However, at deeper depths, the pontoon positive buoyancy is significantly increased due to the loss of mass of the buoyancy gas vented during the ascent. The change in positive buoyancy may be expressed as

$$\Delta F_B = \Delta\rho_g A_p L_p = \frac{\Delta\rho_g}{\rho_w} F_p$$

$$\text{or} \qquad \frac{\Delta F_B}{F_p} = \frac{\Delta\rho_g}{\rho_w} \tag{8}$$

where $\Delta\rho_g$ = change in buoyancy gas density

A hydrazine fueled pontoon, for example, ascending from 20,000 ft would have an initial buoyancy gas density of approximately 14 lb/ft^3 and buoyancy gas density at the surface of approximately 0.03 lb/ft^3. Thus, the increase in positive buoyancy would be

$$\frac{\Delta F_B}{F_p} = \frac{14 - 0.03}{64} = 0.22$$

That is, a recovery system just slightly buoyant at 20,000 ft would have a positive buoyancy of 22% at the surface. Similarly, a recovery system which had 10% positive buoyancy at 20,000 ft would have 32% positive buoyancy at the surface.

The change in ascent velocity over large depth ranges may be expressed as

$$\dot{X}_i + \Delta\dot{X} = \left[\frac{2(F_{Bi} + \Delta F_B)}{\rho_w C_D A_D}\right]^{1/2} \tag{9}$$

where $\dot{X}_i$ = initial terminal velocity

F_{Bi} = initial system positive buoyancy

Rearranging equation (9) and using equation (1) we obtain

$$\begin{aligned}\frac{\Delta\dot{X}}{\dot{X}_i} &= \left[\frac{2(F_{Bi} + \Delta F_B)}{\rho_w C_D A_D}\right]^{1/2}\left(\frac{\rho_w C_D A_D}{2F_{Bi}}\right)^{1/2} - 1\\ &= \left(\frac{F_{Bi} + \Delta F_B}{F_{Bi}}\right)^{1/2} - 1\\ &= \left(1 + \frac{\Delta F_B}{F_p}\frac{FF_p}{F_{Bi}}\right)^{1/2} - 1\\ &= \left(1 + \frac{\Delta\rho_g}{\rho_w}\frac{1}{\beta_i}\right)^{1/2} - 1\end{aligned} \tag{10}$$

where $\beta_i = F_{Bi}/F_p$

The percentage change in ascent velocity is dependent only on the change in buoyancy gas density and the initial buoyancy ratio and is independent of the size of either the pontoon or the object. The change in buoyancy gas density is dependent on the type of buoyancy gas and the recovery depth.

Figure 2 shows the variation of ascent velocity change with initial buoyancy ratio and gas density change for hydrazine produced buoyancy gas at depths of 5000 and 20,000 ft. At an initial buoyancy ratio of 5%, the ascent velocity increase is 140% and 57% from depths of 20,000 ft and 5000 ft, respectively. Thus, recovery operations from deep depths will result in significant increases in ascent velocity as the system ascends through the water column. Also, increasing the initial buoyancy ratio will decrease the percentage of velocity change which will be experienced over a specified depth range.

MAXIMUM PORPOISE HEIGHT

When the pontoon reaches the water surface, the positive buoyancy of the pontoon pushes the top of the pontoon a short distance out of the water. The inertia of the system causes the pontoon to overshoot this equilibrium distance and move farther out of the water. The equation of motion for the pontoon porpoising the surface is

$$M_T \frac{d^2x}{dt^2} = F_{Bs} - F_L \qquad (11)$$

where M_T = total mass of pontoon and object, constant

F_{Bs} = pontoon positive buoyancy before porpoising, constant

F_L = loss of buoyancy due to porpoising

X = distance from top of pontoon to surface

Drag forces acting on the pontoon during porpoising have been neglected. This assumption will yield slightly higher porpoise heights than if a drag force were assumed present but allows equation (11) to be integrated.

The loss of buoyancy due to porpoising, neglecting the pontoon mass, may be expressed as

$$F_L = \rho_W A_P X \qquad (\text{for } X \geq 0) \qquad (12)$$

Substituting equation (12) into (11) and recalling that

$$\frac{d^2X}{dt^2} = \ddot{X} = \frac{d\dot{X}}{dX}\frac{dX}{dt} = \dot{X}\frac{d\dot{X}}{dX}$$

we obtain

$$\dot{X}\frac{d\dot{X}}{dX} = \frac{F_{Bs}}{M_T} - \frac{\rho_W A_P}{M_T} X$$

or

$$\dot{X}d\dot{X} = \frac{F_{Bs}}{M_T}\,dX - \frac{\rho_W A_P}{M_T}\,XdX \tag{13}$$

Integrating equation (13) from $X = 0$ to $X = X_{max}$

and $\dot{X} = \dot{X}_s$ to $\dot{X} = 0$

where X_{max} = maximum porpoise height

$\dot{X}_s$ = ascent velocity at surface before porpoising

we obtain

$$-\frac{\dot{X}_s^2}{2} = \frac{F_{Bs}}{M_T}\,X_{max} - \frac{\rho_W A_P}{M_T}\,\frac{X_{max}^2}{2}$$

or solving for X_{max}

$$X_{max} = \frac{F_{Bs}}{\rho_W A_P} + \left[\left(\frac{F_{Bs}}{\rho_W A_P}\right)^2 + \frac{M_T \dot{X}_s^2}{\rho_W A_P}\right]^{1/2}$$

Dividing through by L_P we obtain

$$\frac{X_{max}}{L_P} = \beta_s + \left[\beta_s^2 + \frac{M_T \dot{X}_s^2}{F_P L_P}\right]^{1/2} \tag{14}$$

where $\beta_s = F_{Bs}/F_P$

Replacing ascent velocity with equation (7) we finally obtain

$$\frac{X_{max}}{L_P} = \beta_s + \left[\beta_s^2 + \frac{2}{C_D}\,\beta_s\,\alpha\,\frac{M_T}{F_P}\right]^{1/2} \tag{15}$$

The nondimensional porpoise height depends on the buoyancy ratio at the surface, the empirical drag coefficient, the pontoon to drag area ratio and the total system mass to pontoon displacement ratio. Neglecting variations in the drag coefficient, and for the special case where the pontoon area is equal to the drag area, the nondimensional porpoise height is independent of the pontoon length-to-diameter ratio. That is, for a given buoyancy ratio, a long slender pontoon will porpoise out of the water proportionally as far as a short stubby one.

Another special case of interest is one where the object area is very large compared to the pontoon area ($\alpha \simeq 0$), then

$$\frac{X_{max}}{L_P} \simeq 2\beta_s \qquad (\alpha \simeq 0) \tag{16}$$

This is the smallest maximum porpoise height which could be realized from equation (15).

For most recovery systems being considered, the pontoon mass is negligible compared to the object mass. Hence

$$M_T \simeq M_o$$

where M_o = object mass

For this case

$$F_P - F_{Bs} = (\rho_o - \rho_w)\, V_o$$

$$= (\rho_o - \rho_w)\, \frac{M_o}{\rho_o}$$

or

$$\frac{M_o}{F_P} = \frac{\rho_o}{\rho_o - \rho_w}\,(1 - \beta_s) \tag{17}$$

where V_o = effective volume of object

ρ_o = effective density of object

Introducing the object specific gravity

$$SG_o = \frac{\rho_o}{\rho_w}$$

equation (17) becomes

$$\frac{M_T}{F_P} \simeq \frac{M_o}{F_P} = \frac{SG_o}{SG_o - 1}\,(1 - \beta_s) \tag{18}$$

Substituting equation (18) into (15) we obtain

$$\frac{X_{max}}{L_p} = \beta_s + \left[\beta_s^2 + \frac{2}{C_D} \beta_s (1 - \beta_s) \left(\frac{SG_o}{SG_o - 1}\right)\alpha \right]^{1/2} \quad (M_T = M_o) \qquad (19)$$

For this case where the recovery system mass is negligible compared to the object mass, the pontoon porpoise height is dependent on the buoyancy ratio at the surface, the empirical drag coefficient, the object specific gravity, and the pontoon-to-drag area ratio. Figure 3 shows the effect of these parameters on the porpoise height for drag coefficients of 1.0 and 1.5. Increasing the surface buoyancy ratio or the pontoon-to-drag area ratio increases the porpoise height, while increasing the object specific gravity or the drag coefficient decreases the porpoise height. From the range of parameters shown in this figure, it is evident that for a typical recovery operation, the pontoon would not porpoise completely out of the water.

SOFT PONTOON CRITICAL RE-ENTRY DEPTH

Once the pontoon has reached its maximum porpoise height, it will re-enter the water and start oscillating at the surface in a damped manner. Buoyancy gas venting, which has occurred continuously during the ascent, ceases at the maximum porpoise height (i.e., minimum ambient pressure) and the mass of buoyancy gas remaining in the pontoon becomes fixed. As the pontoon oscillates at the surface, either differential pressures are created across the pontoon walls or the displacement of pontoon varies. The re-entry depth on the first oscillation will be the largest and the most critical to the pontoon. For hard pontoons, that is pontoons whose displacement does not vary with the differential pressure between the pontoon and the ambient water, the pontoon must be structurally capable of withstanding the differential pressures encountered during this bobbing at the surface. The displacement or positive buoyancy of the pontoon, however, remains constant. For soft pontoons, that is pontoons where increasing water pressure will compress the buoyancy gas in the pontoon, differential pressures are eliminated but pontoon displacement will decrease with increasing depth. The positive buoyancy of the system will be a minimum at the maximum re-entry depth and if the buoyancy decrease due to re-entry exceeds the original net positive buoyancy of the pontoon and object, the system will sink back to the bottom.

At the maximum porpoise height (minimum ambient pressure) the amount of gas in the pontoon becomes fixed. The pressure and volume of the gas at this point is

$$P_1 = Pa + \rho_w (L_p - X_{max}) \qquad (20)$$

$$V_1 = A_p L_p \qquad (21)$$

where P_1 = pontoon gas pressure at maximum porpoise height
V_1 = pontoon gas volume at maximum porpoise height
Pa = atmospheric pressure

At any other time the gas pressure and volume are

$$P = Pa + \rho_w (L_p - X - H) \tag{22}$$

$$V = V_1 - A_p H \tag{23}$$

where H = distance water enters pontoon to compress the gas

Assuming the gas remains at a constant temperature, we may apply Boyle's law and relate equations (20) through (23)

$$\frac{P}{P_1} = \frac{V_1}{V}$$

or

$$\frac{Pa + \rho_w (L_p - X - H)}{Pa + \rho_w (L_p - X_{max})} = \frac{A_p L_p}{A_p (L_p - H)}$$

Solving for X/L_p

$$\frac{X}{L_p} = 1 - \frac{H}{L_p} - \frac{Pa}{\rho_w L_p} \left(\frac{H/L_p}{1 - H/L_p}\right) - \frac{1 - \frac{X_{max}}{L_p}}{1 - H/L_p} \tag{24}$$

The maximum distance the water may enter the pontoon without causing the system to sink is

$$H_{crit} = \frac{F_{Bs}}{\rho_w A_p} \tag{25}$$

Note that equation (25) represents the condition where neutral buoyancy is reached. Also,

$$\frac{H_{crit}}{L} = \beta_s \tag{26}$$

Substituting equation (26) into (24) we obtain

$$\frac{X_{crit}}{L_p} = 1 - \beta_s - \frac{Pa}{\rho_w L_p} \left(\frac{\beta_s}{1 - \beta_s}\right) - \frac{1 - X_{max}/L_p}{1 - \beta_s} \tag{27}$$

where X_{crit} = maximum re-entry depth without sinking

The critical re-entry depth is dependent on the surface buoyancy ratio, the pontoon length, and the maximum porpoise height. Increasing either the pontoon length or the maximum porpoise height decreases the critical re-entry depth, while increasing the surface buoyancy ratio increases it. Figure 4 shows these trends more clearly. The upper curve in Figure 4 shows the variation of re-entry depth with buoyancy ratio for pontoon lengths of 5 and 20 ft with an initial porpoise height of 25% of their respective pontoon lengths. A re-entry depth of 0% means the top of the pontoon can only return to the surface but no further. Negative values of X_{crit}/L_p represent distances below the surface the top of the pontoon may descend to without sinking. A 20-ft long pontoon which has porpoised 25% out of the water and had a buoyancy ratio of 7% before porpoising cannot become fully submerged without sinking. If the buoyancy ratio is increased to 20%, the pontoon may descend approximately one-half a pontoon length below the surface, or about 10 ft. But if the maximum porpoise height is increased to 75%, a 20-ft long pontoon cannot become fully submerged even at a 20% buoyancy ratio. Similarly, a 5-ft long pontoon which has porpoised 75% out of the water must have a buoyancy ratio greater than 9% before it could become fully submerged and still remain at the surface. Thus, it is clearly evident that soft, cylindrical pontoons, more than a few feet in length, are not expected to remain at the surface after porpoising unless additional provisions are provided to control the pontoon motion at the surface.

SAMPLE CASES

To illustrate the use of these equations, three sample cases are presented.

Case I

Consider a concrete and steel object which is to be recovered from 5000 ft. The object has an in-water weight of 9.9 S tons, a specific gravity of 2.25, and a maximum diameter of 6 ft in the horizontal plane. The object is to be recovered with a soft, hydrazine-fueled, cylindrical pontoon of negligible mass which has a displacement of 10 S tons and is 6 ft in diameter and 12 ft long. The overall recovery system drag coefficient is 1.0.

Hence

$$\beta_i = 0.01$$
$$F_{Bi} = 200 \text{ lb}$$
$$\alpha = 1$$
$$C_D = 1$$
$$L_p = 12 \text{ ft}$$
$$SG_o = 2.25$$
$$\rho_w = 64 \text{ lb/ft}^3$$

The initial terminal velocity is (equation (7)):

$$\dot{X}_i = (\frac{2}{C_D} \beta_i \alpha L_p)^{1/2}$$

$$= \left| \frac{2}{1} (0.01)(1)(12)(32.2) \right|^{1/2}$$

$$= 2.78 \text{ ft/sec}$$

The gas density change from 5000 ft to the surface is

$$\frac{\Delta\rho_g}{\rho_w} = \frac{4.69 - 0.03}{64} = 0.0728$$

and the increase in positive buoyancy is (equation (8)):

$$\frac{\Delta F_B}{F_p} = \frac{\Delta\rho_g}{\rho_w} = 0.0728$$

$$\Delta F_B = 0.0728(10)(2000) = 1456 \text{ lb}$$

The buoyancy ratio at the surface is:

$$\beta_s = \frac{F_{Bi} + \Delta F_B}{F_p} = \frac{200 + 1456}{20,000} = 0.0828$$

The corresponding change in ascent velocity is (equation 10)):

$$\frac{\Delta\dot{X}}{\dot{X}_i} = (1 + \frac{\Delta\rho_g}{\rho_w} \frac{1}{\beta_i})^{1/2} - 1$$

$$= \left|1 + (0.0728)(\frac{1}{0.01})\right|^{1/2} - 1$$

$$= 1.878$$

$$\Delta\dot{X} = 1.878(2.78) = 5.22 \text{ ft/sec}$$

The pontoon ascent velocity at the surface just before porpoising is:

$$\dot{X}_s = \dot{X}_i + \Delta\dot{X} = 2.78 + 5.22 = 8.00 \text{ ft/sec}$$

Thus, the recovery system starts up from the bottom with an initial positive buoyancy of 1% and a velocity of 2.8 ft/sec. By the time the system reaches the surface, the positive buoyancy has increased to 8.3% and the ascent velocity is 8 ft/sec.

The maximum porpoise height of the pontoon at the surface is (equation (19)):

$$\frac{X_{max}}{L_P} = \beta_s + \left[\beta_s^2 + \frac{2}{C_D}\ \beta_s(1 - \beta_s)(\frac{SG_o}{SG_o - 1})\alpha\right]^{1/2}$$

$$= 0.0828 + \left[(0.0828)^2 + (\frac{2}{1})(0.0828)(1 - 0.0828)(\frac{2.25}{2.25 - 1})(1)\right]^{1/2}$$

$$= 0.612$$

$$X_{max} = (0.612)(12) = 7.35 \text{ ft}$$

The critical re-entry depth of the pontoon after porpoising is (equation (27)):

$$\frac{X_{crit}}{L_P} = 1 - \beta_s - \frac{P_a}{\rho_W L_P}\ (\frac{\beta_s}{1 - \beta_s}) - \frac{1 - X_{max}/L_P}{1 - \beta_s}$$

$$= 1 - 0.0828 - \frac{(14.7)(144)}{(64)(12)}\ (\frac{0.0828}{1 - 0.0828}) - \frac{1 - 0.612}{1 - 0.0828}$$

$$= + 0.245 \quad (>0)$$

$$X_{crit} = + 2.94 \text{ ft}$$

Since the critical re-entry depth is greater than zero, the pontoon would not remain at the surface after porpoising.

Case II

To limit the rise rate and porpoising, it is decided to add a drogue chute to the pontoon in the previous example. The drogue chute has a diameter of 20 ft and the overall drag coefficient of the system remains at 1.0.

Hence, $\alpha = (\frac{6}{20})^2 = 0.09$, instead of 1.0.

The initial terminal velocity is

$$\dot{X}_i = \left| \frac{2}{1} (0.01)(0.09)(12)(32.2) \right|^{1/2} = 0.83 \text{ ft/sec}$$

The positive buoyancy and velocity increase during the ascent remain the same as in the previous example

$$\beta_s = 0.0828$$

$$\frac{\Delta \dot{X}}{\dot{X}_i} = 1.878$$

The ascent velocity at the surface becomes

$$\dot{X}_s = \dot{X}_i + \Delta\dot{X} = 0.83 + 1.878(0.83) = 2.39 \text{ ft/sec}$$

The maximum porpoise height at the surface is

$$\frac{X_{max}}{L_p} = 0.0828 + \left[(0.0828)^2 + \frac{2}{1}(0.0828)(1 - 0.0828)\left(\frac{2.25}{1.25}\right)(0.09)\right]^{1/2}$$

$$= 0.260$$

$$X_{max} = 3.12 \text{ ft}$$

And, the critical re-entry depth is

$$\frac{X_{crit}}{L_p} = 1 - 0.0828 - \frac{(14.7)(144)}{(64((12)}\left(\frac{0.0828}{1 - 0.0828}\right) - \frac{1 - 0.260}{1 - 0.0828}$$

$$= -0.14$$

$$X_{crit} = -1.66 \text{ ft}$$

Thus, the drogue chute reduced the maximum ascent velocity from 8 ft/sec to 2.4 ft/sec, reduced the porpoise height from 7.4 ft to 3.1 ft, and increased the critical re-entry depth from 2.9 ft above the water surface to 1.7 ft below the surface. But, a pontoon is expected to re-enter the water at least as far as it porpoised out of the water; hence, the pontoon will still not remain at the surface.

Case III

To try to keep the pontoon at the surface it is decided to employ a drogue chute with a higher drag coefficient. In this case, the chute diameter is held at 20 ft but the overall drag coefficient is increased to 4.0.

The initial terminal velocity is

$$\dot{X}_i = \left[\frac{2}{4}\,(0.01)(0.09)(12)(32.2)\right]^{1/2} = 0.42 \text{ ft/sec}$$

The velocity at the surface is

$$\dot{X}_S = 0.42 + 1.878(0.42) = 1.20 \text{ ft/sec}$$

The maximum porpoise height is

$$\frac{X_{max}}{L_p} = (0.0828) + \left[(0.0828)^2 + \frac{2}{4}(0.0828)(1 - 0.0828)\left(\frac{2.25}{1.25}\right)(0.09)\right]^{1/2}$$

$$= 0.197$$

$$X_{max} = 2.36 \text{ ft}$$

The critical re-entry depth becomes

$$\frac{X_{crit}}{L_p} = 1 - 0.0828 - \frac{(14.7)(144)}{(64)(12)}\left(\frac{0.0828}{1 - 0.0828}\right) - \frac{1 - 0.197}{1 - 0.0828}$$

$$= -0.207$$

$$X_{crit} = -2.49 \text{ ft}$$

In this case, the ascent velocity range is between 0.4 ft/sec and 1.2 ft/sec and the excursions at the surface are moderate with a maximum porpoise height of 1.4 ft and a critical re-entry depth of 2.5 ft.

The ability of this system to remain at the surface must be considered marginal since the critical re-entry depth is only slightly greater than the porpoise height.

SUMMARY

1. To limit the ascent rate of large pontoons to 2 ft/sec or less, positive buoyancy ratios of 1% or less may be required. At these low buoyancy ratios slight variations in the flow field may have a profound influence on the motion of the system.

2. Recovery systems ascending from deep depths will experience significant increases in positive buoyancy and rise rates. These increases are the result of buoyancy gas being vented during ascent and thereby decreasing the in-water weight of the system.

3. Pontoon porpoising at the surface is not excessive and will be less than one pontoon length for the vast majority of cases.

4. Soft, cylindrical pontoons, more than a few feet in length, are not expected to remain at the surface after porpoising unless additional provisions are provided to control the pontoon motion at the surface.

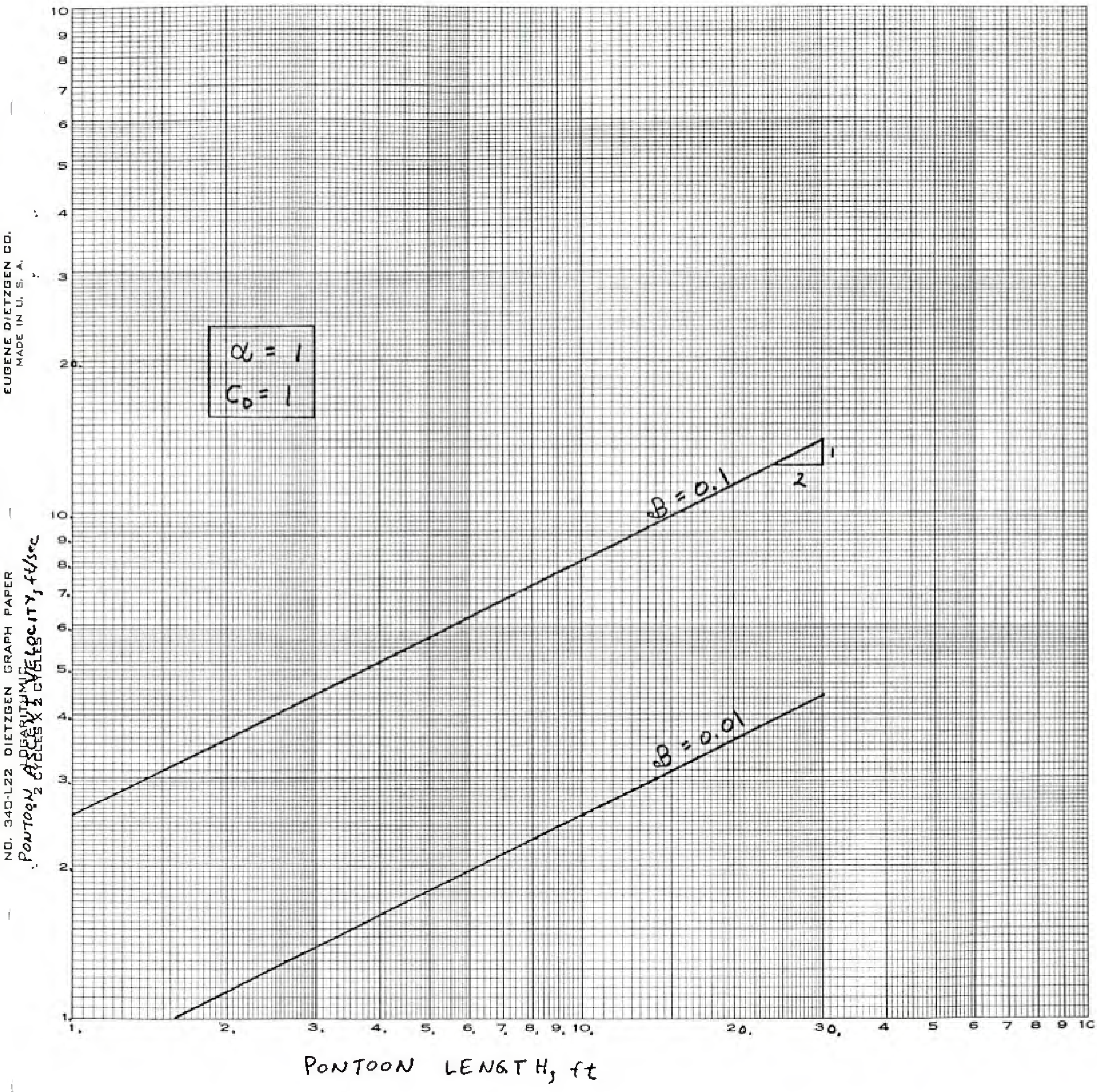


FIGURE 1. VARIATION OF ASCENT VELOCITY WITH PONTOON LENGTH AND BUOYANCY RATIO.

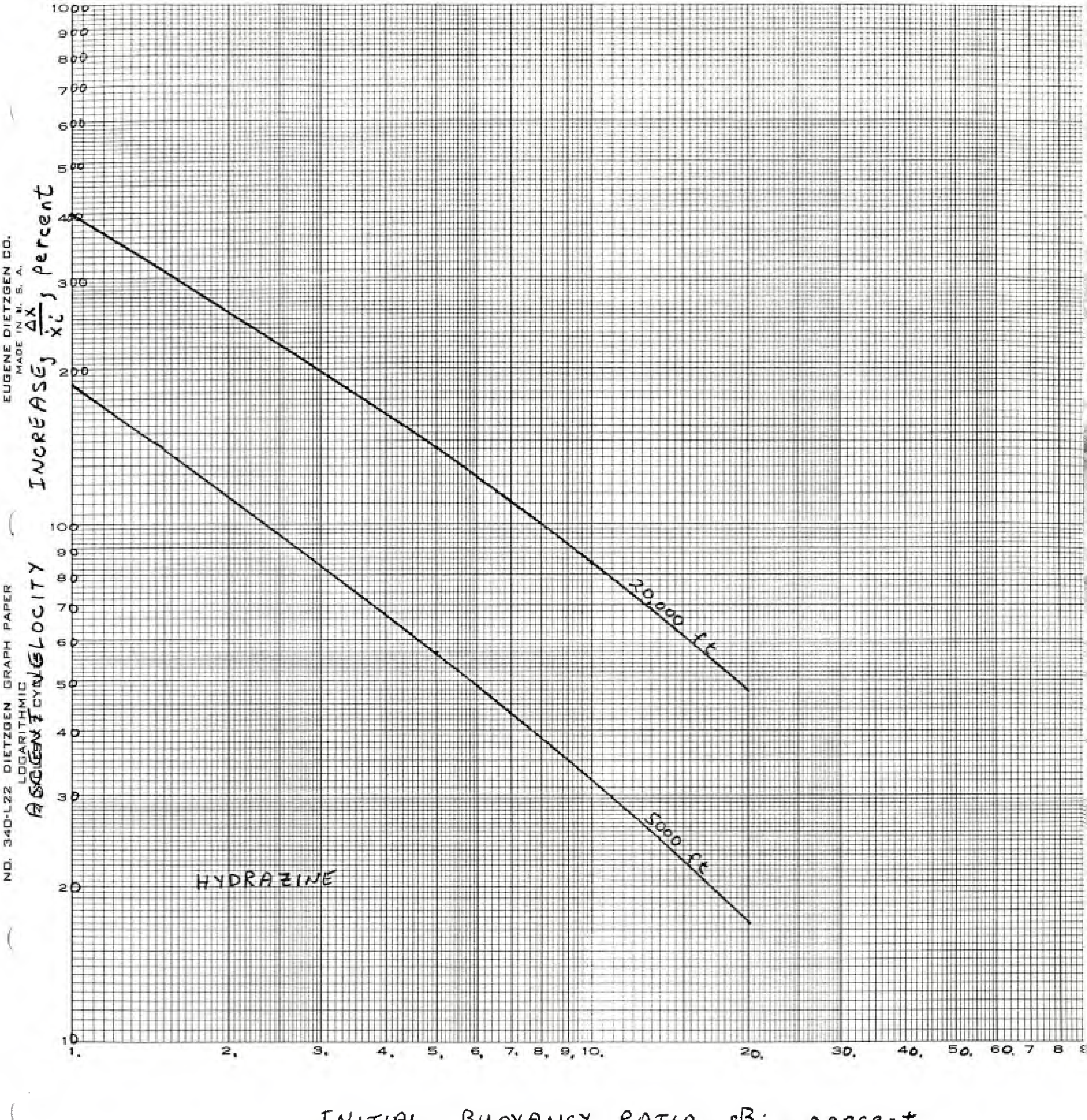


FIGURE 2. VARIATION OF VELOCITY INCREASE WITH INITIAL BUOYANCY RATIO FROM 5000 AND 20,000 ft.

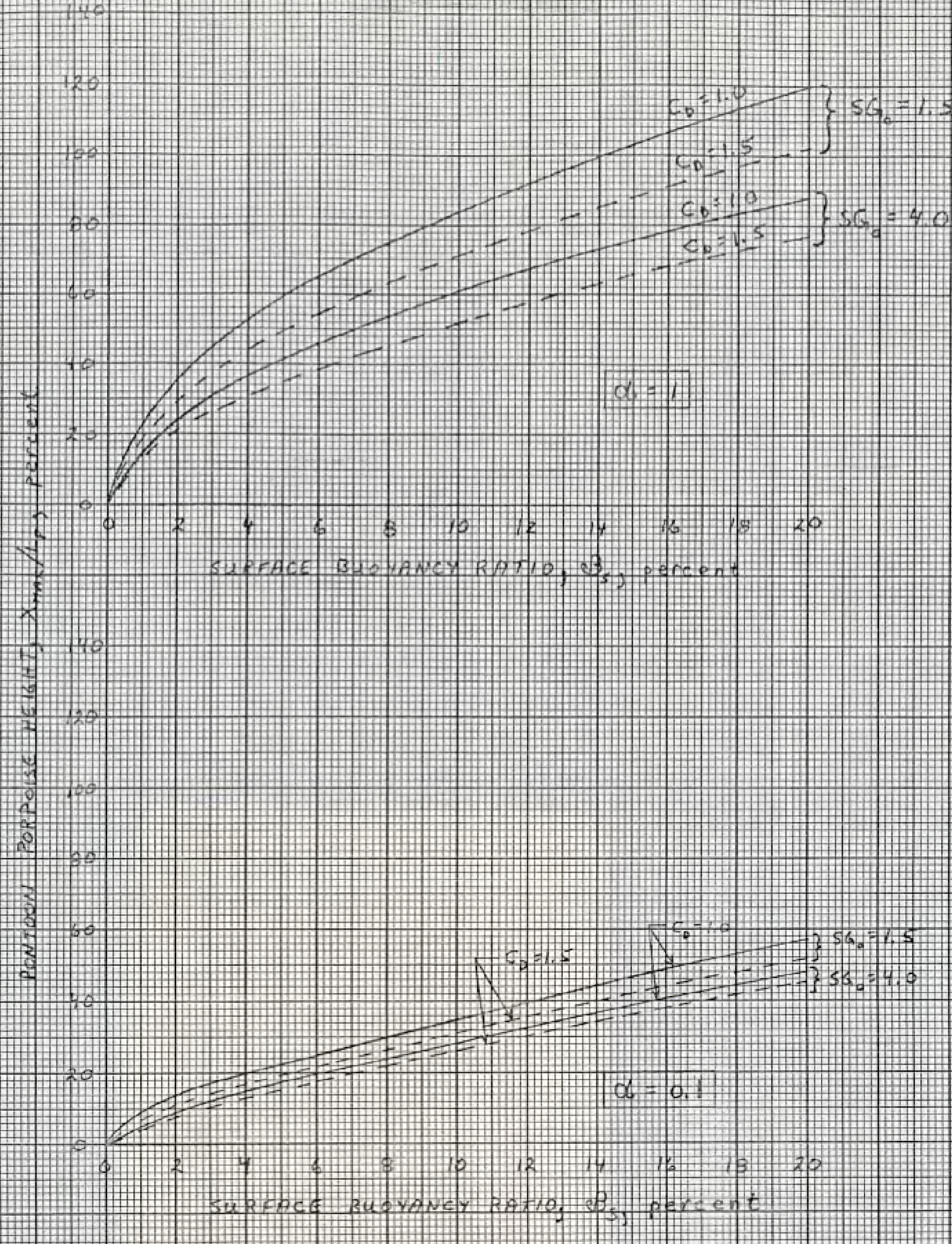


FIGURE 3. VARIATION OF PONTOON PORPOISE HEIGHT WITH BUOYANCY RATIO, DRAG COEFFICIENT, AND OBJECT SPECIFIC GRAVITY.

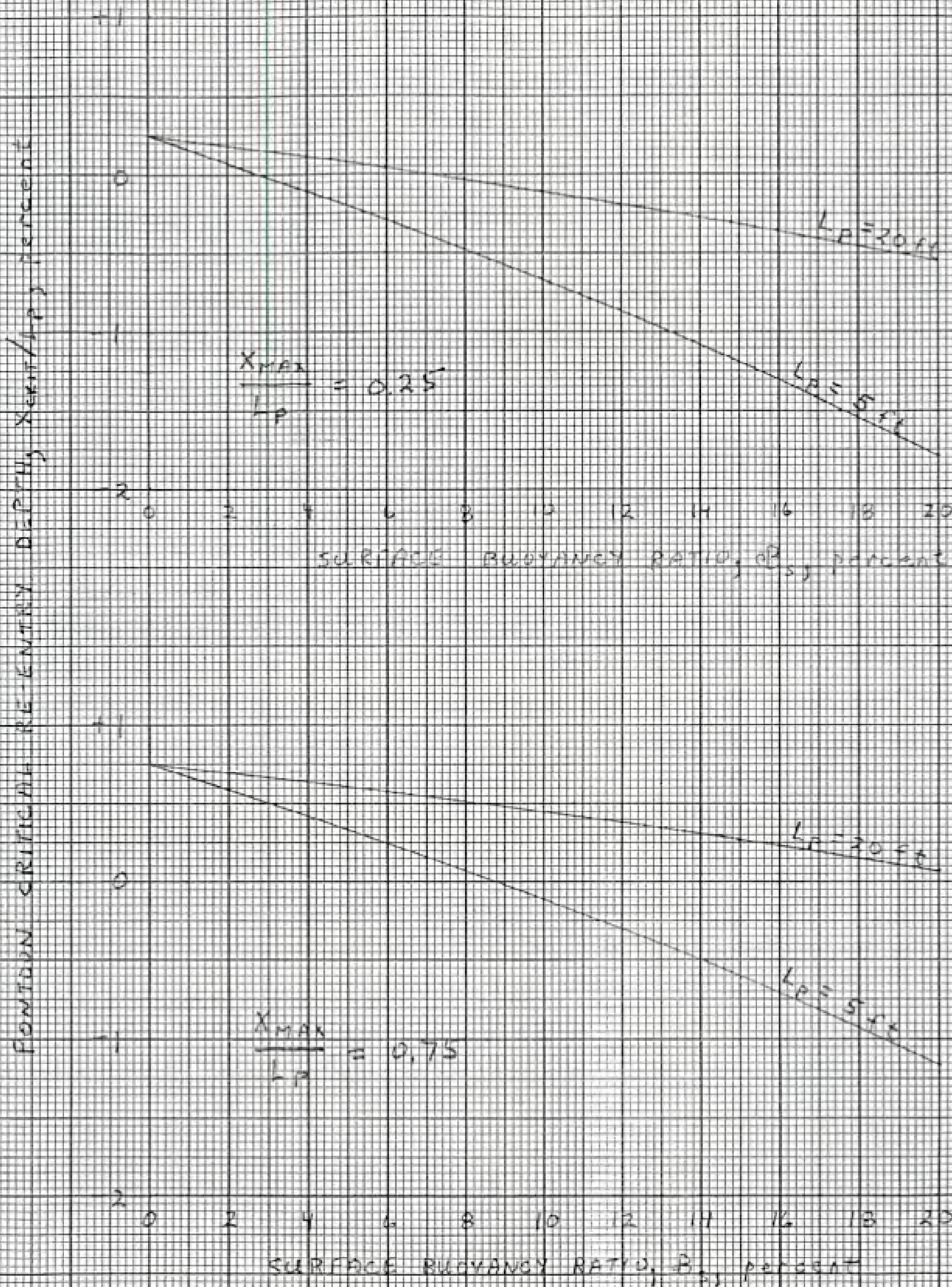


FIGURE 4. VARIATION OF CRITICAL RE-ENTRY DEPTH WITH PONTOON LENGTH AND BUOYANCY RATIO.